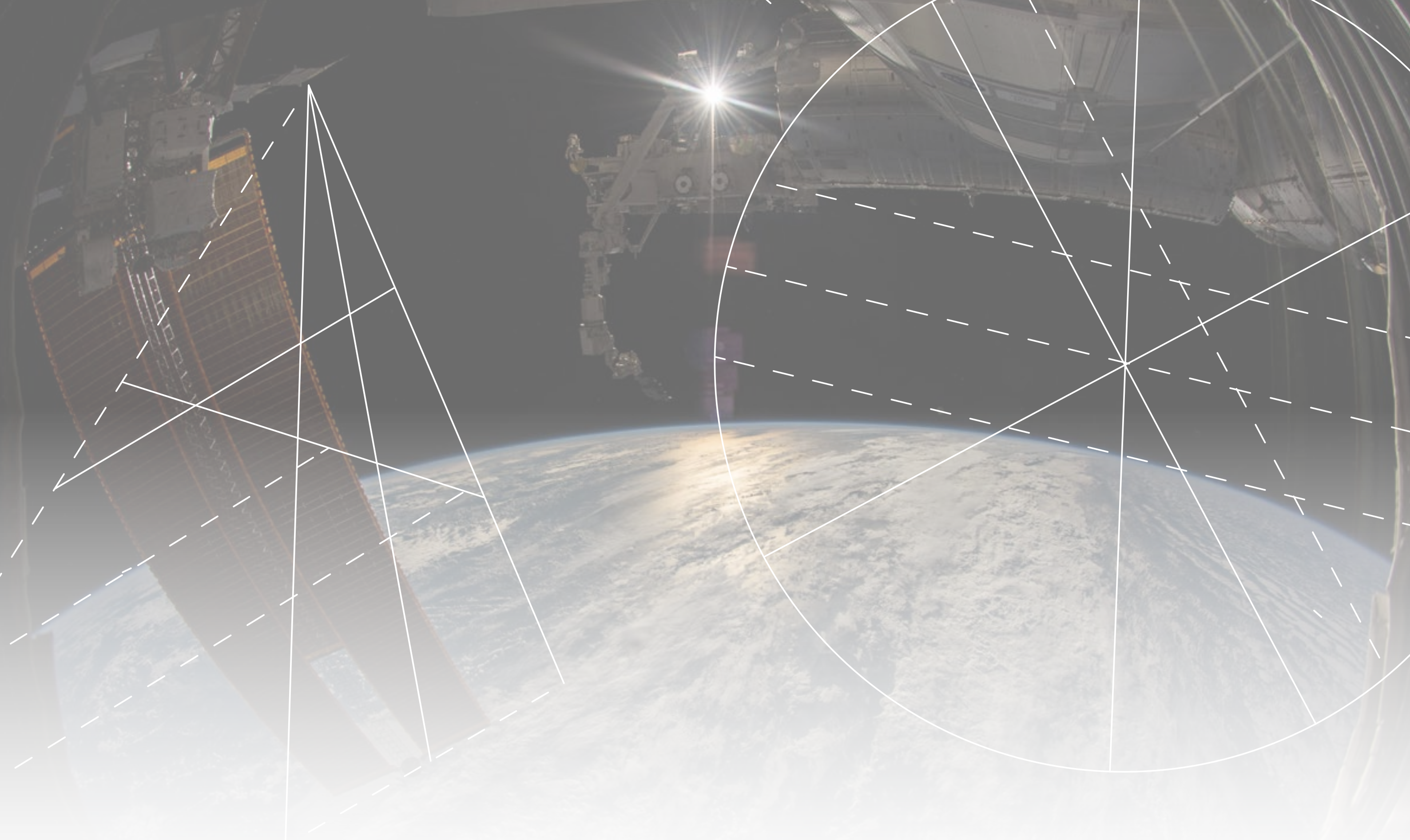


principia mission

Maths in Space Pack 2

Exploring combinations, permutations, factorials and probability surrounding Tim Peake's Mission to the International Space Station (ISS)

A mathematics resource for primary and secondary school teachers

11 - 16

UK SPACE AGENCY

esero

STEM LEARNING

UK Space Agency

The UK Space Agency is at the heart of UK efforts to explore and benefit from space. The UK's thriving space sector contributes £11.8 billion a year to the UK economy and directly employs over 34,000 people with an average growth rate of almost 8.5%.

The UK Space Agency is responsible for ensuring that the UK retains and grows a strategic capability in space-based systems, technologies, science and applications

STEM Learning Ltd

STEM Learning Ltd operates the National STEM Learning Centre and Network; providing support locally, through Science Learning Partnerships across England, and partners in Scotland, Wales and Northern Ireland; alongside a range of other projects supporting STEM education. STEM Learning is an initiative of the White Rose University Consortium (comprising the Universities of Leeds, Sheffield and York) and Sheffield Hallam University.

ESERO

ESERO-UK, also known as the UK Space Education and Resource Office, aims to promote the use of space to enhance and support the teaching and learning of science, technology, engineering and mathematics (STEM) in schools and colleges throughout the UK.

Principia

Tim's mission to the International Space Station, called 'Principia', will use the unique environment of space to run experiments as well as try out new technologies for future human exploration missions. Tim will be the first British ESA astronaut to visit the Space Station where he will spend six months as part of the international crew.

Introduction

On December 15th 2015 European Space Agency astronaut Tim Peake launched on the six month Principia mission to the International Space Station. Principia was named after Isaac Newton's *Naturalis Principia Mathematica*, describing the principal laws of motion and gravity.

The education and inspiration of young people is a core element of the Principia mission. Tim is determined to make Principia an exciting adventure for the younger generation. This resource is part of an extensive education programme to inspire children to pursue STEM subjects.

This collection of mathematics resources is aimed at teachers of key stage 2, 3 and 4 students (age 7 to 16), and is closely linked to elements of the mathematics national curriculums of England, Northern Ireland, Scotland and Wales which can be taught in new and stimulating ways. Children can explore familiar and unfamiliar mathematical ideas relating to Tim's Principia mission, including estimation, measures, combinations, permutations and probability.

This teacher guide, and the resources that accompany it,can be used in a number of different ways:

1. Following the activities in sequence will cover the curriculum links listed within. This might be done as part of a themed week, or over a series of sessions.

2. Teachers can pick and choose which activities, resources and links to use and when – they can be used independently of each other. This might enhance the ways in which space and mathematical topics are currently taught. If teachers have specific challenges in mind that align with their interests and those of the children, learning activities might be selectively chosen.

3. Teachers may wish to present children, in class or as part of an extra-curricular activity, with the activities only.

Click here for more teaching resources and ideas linked to Tim's mission.

Introductory videos

The National STEM Learning Centre online resource collection hosts a variety of Tim Peake related resources, from primary to secondary, covering science, technology and computing topics. As part of our collections the following videos may also form a good introduction to some, if not all, of the maths ideas in this resource:

- Tim Peake (http://stem.org.uk/rxce8)
  Tim talks about studying STEM subjects and how he became an astronaut.
- Tim Peake: Becoming an Astronaut (http://stem.org.uk/rxdex)
  Tim talks about the importance of science skills to be able to work on the International Space Station (ISS).
- Can You Get Fat in Space (http://stem.org.uk/rxcvn)
  As part of the Great British Space Dinner competition, celebrity chef, Heston Blumenthal, asks us the question, "Can you get fat in space?"
- Cooking with Astronauts (http://stem.org.uk/rxcz9)
  Heston Blumenthal describes how preparing food on the ISS is different from that on Earth. Water is used to rehydrate foods and the food cannot be heated with ovens or microwaves.
- Cows in Space (http://stem.org.uk/rxcvo)
  As part of the Great British Space Dinner competition, celebrity chef, Heston Blumenthal, asks the question, "Can you take cows into space?"
- Dinner Party in Space (http://stem.org.uk/rxcvr)
  Heston Blumenthal explains that, in the weightless environment on the International Space Station, you cannot have foods that can float around and get into people eyes and instruments, and you need to drink out of plastic bags, rather than cups.
- Food Texture (http://stem.org.uk/rxcvp)
  Heston Blumenthal asks children to think about textures of food for astronauts. He suggests mixing textures together to give the best experience for Tim when he eats his meal.
- Tim Peake's Food Likes (http://stem.org.uk/rxcvq)
  As part of the Great British Space Dinner competition, celebrity chef, Heston Blumenthal, asks astronaut Tim Peake about what foods he likes to eat.

Curriculum Links

Subject content:

- understand and use place value for decimals, measures and integers of any size
- use conventional notation for the priority of operations, including brackets, powers, roots and reciprocals
- work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and or 0.375 and)
- round numbers and measures to an appropriate degree of accuracy [for example, to a number of decimal places or significant figures]
- use a calculator and other technologies to calculate results accurately and then interpret them appropriately
- substitute numerical values into formulae and expressions, including scientific formulae
- understand that the probabilities of all possible outcomes sum to 1
- generate theoretical sample spaces for single and combined events with equally likely, mutually exclusive outcomes and use these to calculate theoretical probabilities.

England:

Working Mathematically:

- select and use appropriate calculation strategies to solve increasingly complex problems
- use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships
- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- make and test conjectures about patterns and relationships; look for proofs or counter-examples
- interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning
- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems.

Wales:

- transfer mathematical skills across the curriculum in a variety of contexts and everyday situations
- select, trial and evaluate a variety of possible approaches and break complex problems into a series of tasks
- prioritise and organise the relevant steps needed to complete the task or reach a solution
- choose an appropriate mental or written strategy and know when it is appropriate to use a calculator
- explain results and procedures precisely using appropriate mathematical language
- interpret answers within the context of the problem and consider whether answers, including calculator, analogue and digital displays, are sensible

Scotland:

- I can use a variety of methods to solve number problems in familiar contexts, clearly communicating my processes and solutions
- I can solve problems by carrying out calculations with a wide range of fractions, decimal fractions and percentages, using my answers to make comparisons and informed choices for real-life situations.
- By applying my understanding of probability, I can determine how many times I expect an event to occur, and use this information to make predictions, risk assessment, informed choices and decisions.

Northern Ireland:

- Examine the role of mathematics as a "key" to entry for future education, training and employment. Explore how the skills developed through mathematics will be useful to a range of careers
- Decide on the appropriate method and equipment to solve problems–mental, written, calculator, mathematical instruments or a combination of these
- Show deeper mathematical understanding by thinking critically and flexibly, solving problems and making informed decisions
- Work effectively with others
- Demonstrate self-management by working systematically, persisting with tasks, evaluating and improving own performance

Teacher Information Resource 1:

Starter, Main, Pudding

This resource looks at combinatorics, a similar process to that used to solve probability problems through listing outcomes or constructing space diagrams that contain all of the possible outcomes when combining different events.

The activity can be introduced with a discussion about which foods learners would take into space. The 'Tim Peake's Food Likes' video (http://stem.org.uk/rxcvq) shows Tim discussing his preferences.

This lesson explores the statement *"On the International Space Station (ISS) food supply is pretty limited. Tim could have just three choices for each part of his meal."* Teachers may wish to discuss whether this is a realistic model - do students think this is the case?

Questions to prompt thinking about the above scenario include:

What three-course meal would you choose?

Discuss that there are a number of different options, depending on preference. How many possible combinations are possible?

Can you list all of the possible combinations?

Do students have a method for approaching this question? Can they set their working out systematically? Is there more than one way of working out the answer?

The total number of combinations is: $3 \times 3 \times 3 = 3^3 = 27$

What if there were only two options for each course?

A list of all possible outcomes with just two options could be made to find the solution. Alternatively the total number of combinations can be calculated as:
$2 \times 2 \times 2 = 2^3 = 8$. Which method do learners think is best? What are the pros and cons for each when applied to similar situations?

What if there were four options?

Again a list of all possible outcomes with four options could be made. The total number of combinations can be calculated as: $4 \times 4 \times 4 = 4^3 = 64$. Can learners again talk about, with reasons, which method they prefer?

Tim is on the ISS for six months, are there enough different combinations for every meal?

This is a more challenging problem. First, how many meals would you expect Tim to eat during a six month period? Does that include breakfast, lunch and dinner? Should we stick with the three course meal plan?
A quick solution to the problem will be to state that with three options per course, this gives $3 \times 3 \times 3 = 3^3 = 27$ different meals. Just under a month, or nine days if every meal must follow the same design. So no, there are not enough combinations to have a different meal over a six month period.
There are roughly 180 days in the period we are interested in, Tim's mission actually lasts for 171 days. If there are n options, to have a different meal each day requires $n^3 > 180$. This could be solved a number of ways, older students may wish to use logarithms, others could attempt trial and improvement.
If there are 6 options ($n = 6$) there will be 216 different combinations. If we wanted a different three course meal, three times a day, for six months, we will need at least 540 meal combinations. When $n = 9$, this gives 729 possible combinations.

Teacher Information Resource 2: Four options

This resource looks at the factorial function (!).The factorial function multiplies consecutive descending numbers, ending in 1. e.g. 4! = 4 × 3 × 2 × 1 = 24

The activity could be introduced with a discussion about what food learners would take into space. The 'Tim Peake's Food Likes' video (http://stem.org.uk/rxcvq) sees Tim discuss his preferences.

This lesson explores looks at arranging four different food options. Teachers may wish to discuss whether this is a realistic model - do students think this is a likely scenario aboard the ISS?

Questions to prompt thinking about this scenario include:

Which option would you choose?

Discuss that there are a number of different options, depending on preference. What would be students' preferred four meals? Is there anything they might miss if they were in space for six months? How do they think food is served in space?

Order the options, from favourite to least favourite

Do learners agree with each other? Can they find people with the same or opposite orders to themselves?

In how many different ways can the four meals be arranged?

There are 24 ways of ordering the four options.
There are four options to select from when choosing the favourite food choice. Once that has been chosen there are three options to choose from fro second place. There are then two options for the third placed option and the one remaining option must be the least favourite.
Note this can be expressed as 4!= 4 ×3 ×2 ×1=24, although teachers may want to refrain from defining the factorial function at this stage.

What if there were a different number of meal options?

Encourage students to think of their own ideas for solving this question. They may need to be prompted into considering the simplest option of just one meal choice, then two, then three options. Can they spot a pattern is the sequence of numbers and predict further results.

Is there a general pattern?

Number of meals	Calculation	Factorial notation	Number of options
1	1	1!	1
2	2 x 1	2!	2
3	3 x 2 x 1	3!	6
4	4 x 3 x 2 x 1	4!	24
5	5 x 4 x 3 x 2 x 1	5!	120
6	6 x 5 x 4 x 3 x 2 x 1	6!	720

Students may describe the term to term rule that you 'multiply by one more each time'. As this stage the factorial function notation could be discussed.

If there were n different meal options the calculation would look like this:

$n! = n \times (n-1) \times (n-2) \ldots \times 2 \times 1$

Further points for discussion:

- What is the first factorial to be larger than 1 000? Larger than 1 000 000?
- What is 0!
- What is the largest factorial your calculator can calculate?
- Can you have negative factorials?

Teacher Information Resource 3: No labels

This resource looks at the probability involved when selecting objects at random. The activity could be introduced with a discussion about how food is transported and eaten in space. The Cooking with Astronauts (http://stem.org.uk/rxcz9) sees Heston Blumenthal describes how water is used to rehydrate food sachets on the ISS.

This lesson explores the statement "Unfortunately for Tim all of his meals have come in packets...but with no labels on them! Tim has eight possible different options to choose from but he doesn't know what is in each pack he selects. He also knows one of the options is curry"

Questions to prompt thinking about this scenario include:

What is the probability of selecting curry?

Students should give the answer of 1/8.

This assumes that only one of the options is curry, that the selection is completely at random and he cannot deduce the content of the packages by weight or consistency.

What is the probability of not choosing curry?

Students should give the answer of 7/8.

Again the same assumptions as above are made. Teachers may wish to highlight that there are seven options that are not curry, out of eight. Alternatively the answer can be calculated by subtracting the probability of selecting curry from one (as the probability of an event happening added to the probability of the event not happening makes one, as it is certain that one of these options will take place).

What is the probability of not choosing curry three different meal times in a row?

Teachers may wish to encourage students to draw a tree diagram to represent this problem. Considering the first meal, the probability of not selecting the curry sachet is 7/8. Assuming that the curry option has not been selected the following meal time the probability is 6/7. For the third meal this becomes 5/6.

As all of these outcome have to happen we multiply, hence the probability of selecting three meals which are not curry is:

$$\frac{7}{8} \times \frac{6}{7} \times \frac{5}{6} \times \frac{5}{8} = 0.625$$

Teachers may also wish to discuss methods for fraction multiplication, including 'cancelling down' before multiplying.

Teacher Information Resource 4: no labels - unlimited options

This resource looks at the probability involved when selecting objects at random, without replacement. The activity could be introduced with a discussion about how food is transported and eaten in space. The Cooking with Astronauts (http://stem.org.uk/rxcz9) sees Heston Blumenthal describes how water is used to rehydrate food sachets on the ISS.

This lesson explores the statement *"Unfortunately for Tim all of his meals have come in packets... but with no labels attached! Tim knows there are eight possible different options, but he doesn't know what is in each pack he selects. Up on the ISS they seem to have an unlimited number of each of the 8 meals choices. He also knows one of the options is curry."*

Questions to prompt thinking about this scenario include:

What is the probability of selecting curry?

Students should give the answer of 1/8.

This assumes that only one of the options is curry, that the selection is completely at random and he cannot deduce the content of the packages by weight or consistency.

What is the probability of not choosing curry?

Students should give the answer of 7/8.

Again the same assumptions as above are made. Teachers may wish to highlight that there are seven options that are not curry, out of eight. Alternatively the answer can be calculated by subtracting the probability of selecting curry from one (as the probability of an event happening added to the probability of the event not happening makes one, as it is certain that one of these options will take place).

What is the probability of not choosing curry three different meal times in a row?

Teachers may wish to encourage students to draw a tree diagram to represent this problem.

The probability of not selecting the curry sachet is 7/8 for the first meal. Assuming that there are an unlimited amounts of each option, the probability of not selecting curry the following remains 7/8. This probability remains for all subsequent meals.

Hence the probability of selecting three meals which are not curry is:

$$\frac{7}{8} \times \frac{7}{8} \times \frac{7}{8} = \frac{343}{512} = 0.669921875 = 0.67 \text{ (2 s.f.)}$$

Teachers may wish to discuss why methods for 'cancelling down' this particular fraction multiplication do not work.

Teachers may also wish to discuss the plausibility of this model. Is it likely that there are unlimited food supplies in the ISS? Are there other scenarios which this model could be applied to? Do students believe that the probability of 0.67 is a reasonable estimation?

Teacher Information Resource 5: Combination meals

This resource looks at combinations in mathematics.
The activity could be introduced with a discussion about what food learners would take into space. The 'Tim Peake's Food Likes' video (http://stem.org.uk/rxcvq) sees Tim discuss his preferences.

The lesson explores choosing two items from four different food options.

Questions to prompt thinking about this scenario include:

Which two options would you choose?

Discuss that there are a number of different options, depending on preference. What would be students' preferred meal combination? Is there anything they might miss if they were in space for six months? How do they think food is served in space?

How many combinations are there?

Can students list all of the possible combinations? Are there different methods within the class? Do students have a preferred method?

Do you think the order matters?

Is 'Soup and Burger' the same as 'Burger and Soup'? This is a key discussion. For combinations, the order isn't important (if it is, then these types of situations are called permutations- see the next resource).

What if you were choosing two options from FIVE items? Four items from five?

This question means to encourage students to start looking for ways to find a general pattern. Instead of five options, teachers may want to prompt student to think simplifying the problem to just one or two options, and build up the level of difficulty. Paired or group work and discussion may provide a good basis for this task.

Can you spot any patterns?

The table below gives some values that students may have calculated:

Options	Items selected	Notation	Calculation	Number of combinations when selecting
2	0	$\binom{2}{0}$	$\frac{2!}{0!\,2!}$	1
2	1	$\binom{2}{1}$	$\frac{2!}{1!\,1!}$	2
2	2	$\binom{2}{2}$	$\frac{2!}{2!\,0!}$	1
3	0	$\binom{3}{0}$	$\frac{3!}{0!\,3!}$	1
3	1	$\binom{3}{1}$	$\frac{3!}{1!\,2!}$	3
3	2	$\binom{3}{2}$	$\frac{3!}{2!\,1!}$	3
3	3	$\binom{3}{3}$	$\frac{3!}{3!\,1!}$	1
4	0	$\binom{4}{0}$	$\frac{4!}{0!\,4!}$	1
4	1	$\binom{4}{1}$	$\frac{4!}{1!\,3!}$	4
4	2	$\binom{4}{2}$	$\frac{4!}{2!\,2!}$	6
4	3	$\binom{4}{3}$	$\frac{4!}{3!\,1!}$	4
4	4	$\binom{4}{4}$	$\frac{4!}{4!\,0!}$	1
5	0	$\binom{5}{0}$	$\frac{5!}{0!\,5!}$	1
5	1	$\binom{5}{1}$	$\frac{5!}{1!\,4!}$	5
5	2	$\binom{5}{2}$	$\frac{5!}{2!\,3!}$	10
5	3	$\binom{5}{3}$	$\frac{5!}{3!\,2!}$	10
5	4	$\binom{5}{4}$	$\frac{5!}{4!\,1!}$	5
5	5	$\binom{5}{5}$	$\frac{5!}{5!\,0!}$	1

Is there a general rule?

Teachers may wish to introduce Pascal's triangle in order for patterns to reveal themselves. Students may have attempted to find the values in the table above by listing combinations, teachers may want to consider at what stage to introduce the formula for the number of possible combinations when choosing r objects from n options, given below:

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)(n-2)...(n-r+1)}{r!}$$

Teacher Information Resource 6: Drink menu

This resource looks at the probability involved when selecting objects at random.

The activity could be introduced with a discussion about how food is transported and eaten in space. Dinner Party in Space (http://stem.org.uk/rxcvr) sees Heston Blumenthal explain that you need to drink out of plastic bags, rather than cups on the ISS.

This lesson explores the statement *"Liquid is a vital commodity on the ISS. Tim is down to his last eight drink pouches. He has five orange flavoured, one apple, one cranberry and one grape. He is going to select one at random."*

Questions to prompt thinking about this scenario include:

What is the probability of selecting the grape drink?

Students should give the answer of 1/8.
This assumes that only one of the options is grape, that the selection is completely at random and he cannot deduce the content of the packages by weight or consistency.

What is the probability of selecting orange?

Students should give the answer of 5/8.
Again the same assumptions as above are made. Teachers may wish to highlight that there are five orange flavoured drinks, out of eight, and the probability of not selecting orange is 3/8, and that these two probabilities sum to one.

What is the probability of selecting orange three times in a row?

Teachers may wish to encourage students to draw a tree diagram to represent this problem.

Considering the first selection, the probability of selecting orange is 5/8. Assuming that one orange option has been selected the following probability is 4/7. For the third selection this becomes 3/6.

Hence the probability of selecting three meals which are not curry is:

$$\frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} = \frac{5}{28} = 0.17857... = 0.18 \text{ (2 s.f.)}$$

Teachers may wish to discuss methods for fraction multiplication, including 'cancelling down' before multiplying, as well as the repeating nature of the irrational decimal answer.

Teacher Information Resource 7: Drink menu identical elements

This resource looks at the probability involved when selecting objects at random, when some of the objects are identical.

The activity could be introduced with a discussion about how food is transported and eaten in space. Dinner Party in Space (http://stem.org.uk/rxcvr) sees Heston Blumenthal explain that you need to drink out of plastic bags, rather than cups on the ISS.

This lesson explores the statement “Liquid is a vital commodity on the ISS. Tim is down to his last five drink pouches. He has two orange flavoured, one apple, one cranberry and one grape. He is going to select one at random.”

Questions to prompt thinking about this scenario include:

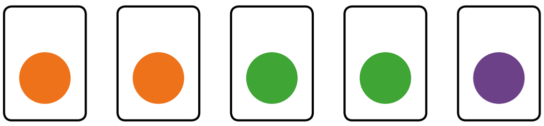
How many different ways of arranging the drinks are there?

Students should give the answer of 60.
If we label the drinks as O1, O2, A, C and G there are 5! = 120 different ways of arranging the drink pouches. As O1 and O2 are identical, each of the 120 arrangements can be matched up as identical pairs. Hence 60 different arrangements.

Teachers may wish to allow student to explore other similar scenarios with different values, before revealing:

“The rule for the number of arrangements a set of n objects, where r of them are identical is $\frac{n}{r}$”

How many arrangements are there for the drink pouches below?



Students should give the answer of 30.
From the statement above students may be able to extend the rule for situations involving more than more set of identical objects. i.e:

“The rule for the number of arrangements a set of n objects, where r objects are identical, s objects are identical and t objects are identical is $\frac{n!}{r!s!t!}$”

This problem becomes $\frac{5!}{2!2!} = \frac{5x4x3x2x1}{2x1x2x1} = 30$

How many different combinations can you make?

On the International Space Station (ISS) food supply is pretty limited. Tim could have just three choices for each part of his meal.

Three starters:	Three mains:	Three puddings:
Soup Olives Melon	Curry Pasta Burger	Popping Candy Ice-lolly Cake

What three-course meal would you choose?

Can you list all of the possible combinations?

What if there were only two options for each course?

What if there were four options?

Tim is on the ISS for 6 months, are there enough different combinations for every meal?

Ordering the meals

Tim has the choice of four items to eat for his main meal each day:

Soup	Burger	Pasta	Curry

Factorials (!)

The factorial function (symbol: !) multiplies a series of descending numbers, ending in 1.
e.g. 4! = 4 x 3 x 2 x 1 = 24

Can you find the factorial button on you calculator?

Try calculating:
3! 5! 7! 1!

Which option would you choose?

Can you order the options, favourite to least favourite?

How many different orders can the four options be arranged?

What if there were only 3 options?

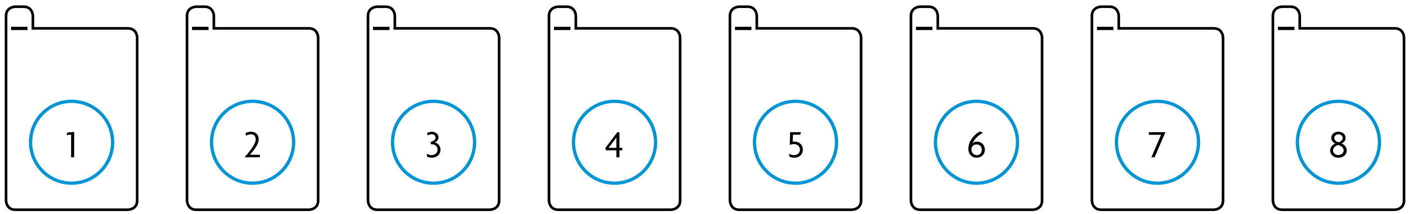
What if there were five options?

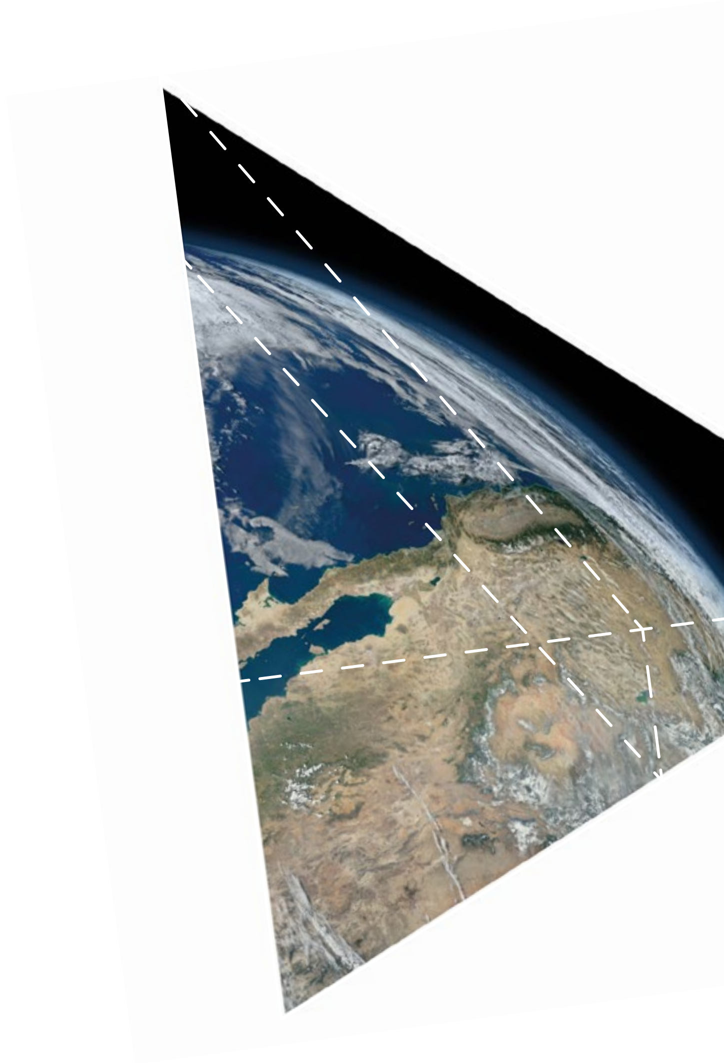
Is there a general pattern?

No labels | Probability

What is likely to happen when a meal is selected?

Unfortunately for Tim all of his meals have come in packets... but with no labels attached!
Tim has eight possible different options to choose from but he doesn't know what is in each pack he selects.
He also knows one of the options is curry.





What is the probability of selecting curry?

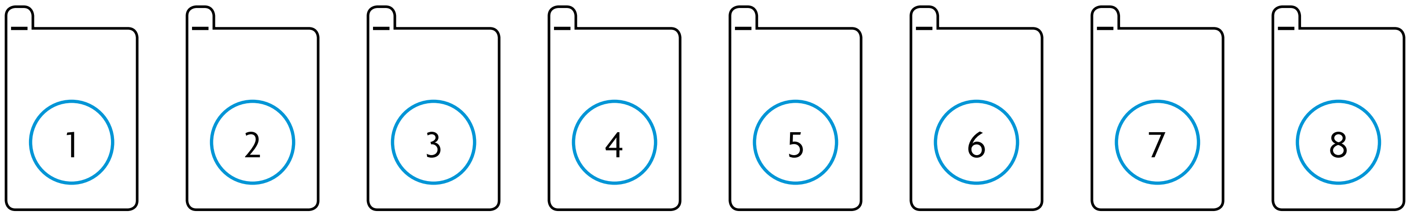
What is the probability of not choosing curry?

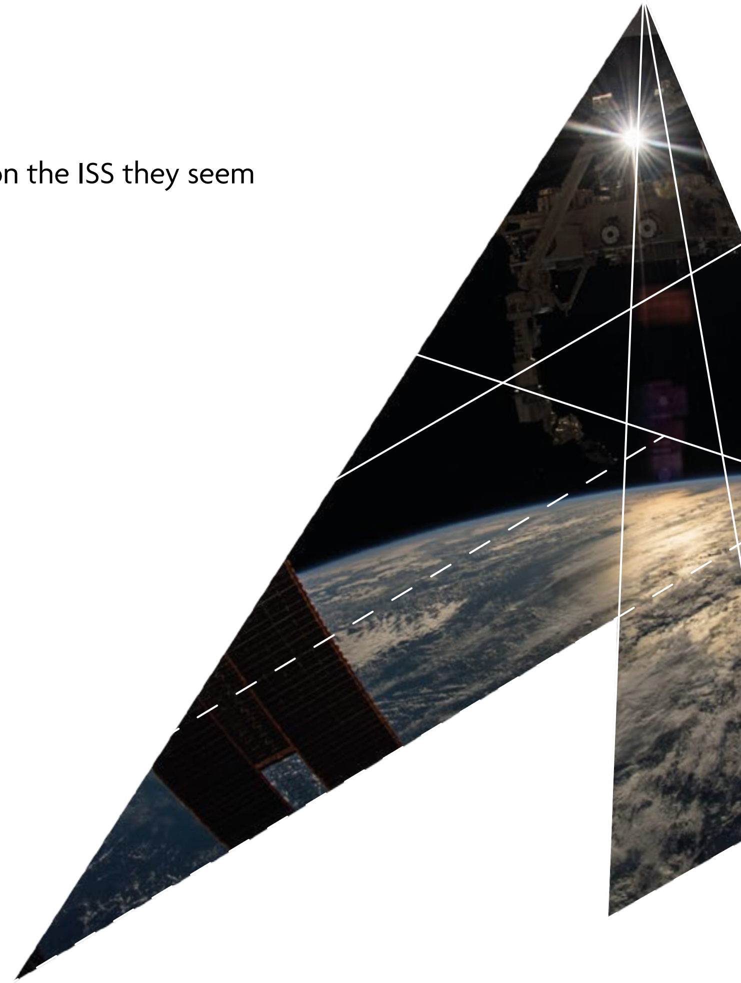
What is the probability of not choosing curry three different meal times in a row?

No labels | Unlimited options

What is likely to happen when a meal is selected?

Unfortunately for Tim all of his meals have come in packets... but with no labels attached!
Tim knows there are eight possible different options, but he doesn't know what is in each pack he selects. Up on the ISS they seem to have an unlimited number of each of the 8 meals choices. He also knows one of the options is curry.





What is the probability of selecting curry?

What is the probability of choosing curry three times in a row?

What is the probability of choosing three different options in a row?

Combination meals

How many combinations are possible?

One meal time Tim can choose two items from the four options:

Soup	Burger	Pasta	Curry

What two options would you choose?

How many combinations are there?

Do you think the order matters?

What if you were choosing two options from FIVE items?

What about three from five items? Four items from five?

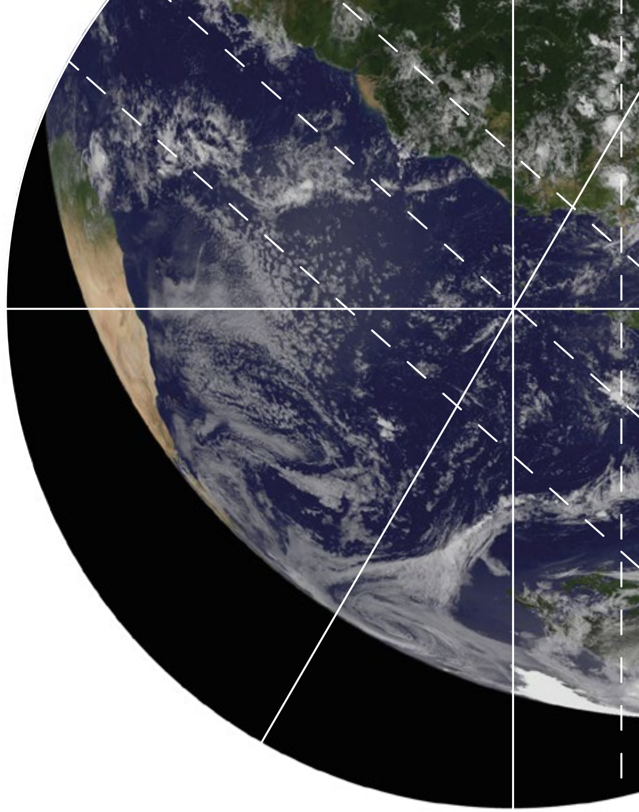
Can you spot any patterns?

Is there a general rule?

Combinations

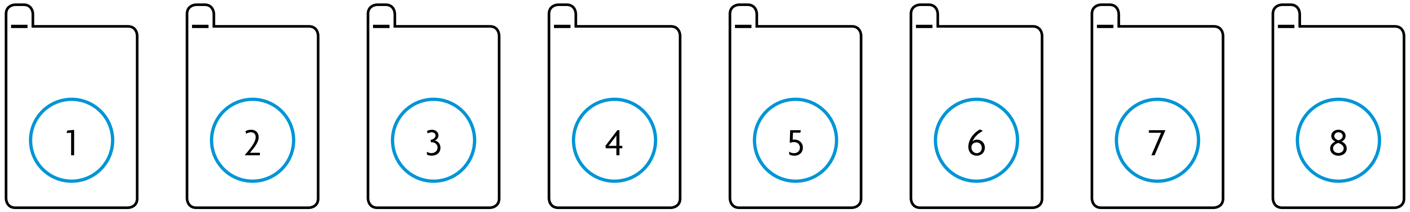
The formula for the number of possible combinations when choosing r objects from n options is given by:

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}$$



What is likely to happen when a meal is selected?

Liquid is a vital commodity on the ISS. Tim is down to his last eight drink pouches.
He has five orange flavoured, one apple, one cranberry and one grape. He is going to select one at random.



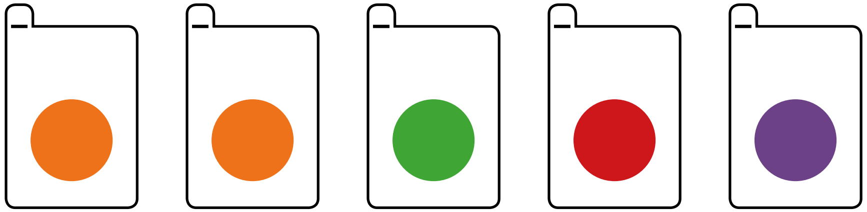
What is the probability of selecting the grape drink?

What is the probability of selecting orange?

What is the probability of selecting orange three times in a row?

What is likely to happen when a meal is selected?

Liquid is a vital commodity on the ISS. Tim is down to his last five drink pouches.
He has two orange flavoured, one apple, one cranberry and one grape.

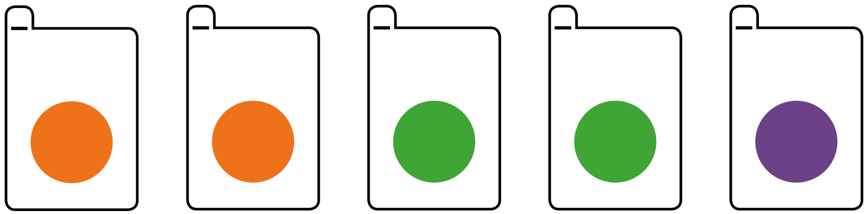


The rule for the number of arrangements a set of n objects, where r of them are identical is $\frac{n!}{r!}$

How many different ways of arranging the drinks are there?

Arrangements continued

How many arrangements are there for the drink pouches below?



The rule for the number of arrangements a set of n objects, where r of them are identical is $\frac{n!}{r!}$

principia mission